nag_tsa_spectrum_univar (g13cbc)

1. Purpose

nag_tsa_spectrum_univar (g13cbc) calculates the smoothed sample spectrum of a univariate time series using spectral smoothing by the trapezium frequency (Daniell) window.

2. Specification

```
#include <nag.h>
#include <nagg13.h>

void nag_tsa_spectrum_univar(Integer nx, NagMeanOrTrend mt_correction,
             double px, Integer mw, double pw, Integer l, Integer kc,
             Nag_LoggedSpectra lg_spect, double x[], double **g,
             Integer *ng, double stats[], NagError *fail)
```

3. Description

The supplied time series may be mean or trend corrected (by least-squares), and tapered, the tapering factors being those of the split cosine bell:

$$\begin{array}{ll} \frac{1}{2}\left(1-\cos\left(\frac{\pi(t-\frac{1}{2})}{T}\right)\right), & 1 \le t \le T \\ \frac{1}{2}\left(1-\cos\left(\frac{\pi(n-t+\frac{1}{2})}{T}\right)\right), & n+1-T \le t \le n \\ 1, & \text{otherwise} \end{array}$$

where $T = \left[\frac{np}{2}\right]$ and p is the tapering proportion.

The unsmoothed sample spectrum

$$f^*(\omega) = \frac{1}{2}\pi \left| \sum_{t=1}^{n} x_t \exp(i\omega t) \right|^2$$

is then calculated for frequency values

$$\omega_k = \frac{2\pi k}{K}\ ,\ k = 0, 1, \ldots, [K/2]$$

where [ ] denotes the integer part.

The smoothed spectrum is returned as a subset of these frequencies for which K is a multiple of a chosen value r, i.e.,

$$\omega_{rl} = \nu_l = \frac{2\pi l}{L},\ \ l = 0, 1, \ldots, [L/2]$$

where $K = r \times L$. The user will normally fix L first, then choose r so that K is sufficiently large to provide an adequate representation for the unsmoothed spectrum, i.e., $K \ge 2 \times n$. It is possible to take $L = K$, i.e., $r = 1$.

The smoothing is defined by a trapezium window whose shape is supplied by the function

$$\begin{array}{ll} W(\alpha) = 1, & |\alpha| \le p \\ W(\alpha) = \frac{1-|\alpha|}{1-p}, & p < |\alpha| \le 1 \end{array}$$

the proportion p being supplied by the user.

The width of the window is fixed as $2\pi/M$ by the user supplying M. A set of averaging weights are constructed:

$$W_k = g \times W\left(\frac{\omega_k M}{\pi}\right)\ ,\ 0 \leq \omega_k \leq \frac{\pi}{M}$$

where g is a normalising constant, and the smoothed spectrum obtained is

$$\hat{f}(\nu_l) = \sum_{|\omega_k| < \frac{\pi}{M}} W_k f^*(\nu_l + \omega_k).$$

If no smoothing is required M should be set to n, in which case the values returned are $\hat{f}(\nu_l) = f^*(\nu_l)$. Otherwise, in order that the smoothing approximates well to an integration, it is essential that $K \gg M$, and preferable, but not essential, that K be a multiple of M. A choice of $L > M$ would normally be required to supply an adequate description of the smoothed spectrum. Typical choices of $L \simeq n$ and $K \simeq 4n$ should be adequate for usual smoothing situations when $M < n/5$.

The sampling distribution of $\hat{f}(\omega)$ is approximately that of a scaled χ^2_d variate, whose degrees of freedom d is provided by the routine, together with multiplying limits mu, ml from which approximate 95% confidence intervals for the true spectrum $f(\omega)$ may be constructed as $[ml \times \hat{f}(\omega), mu \times \hat{f}(\omega)]$. Alternatively, $\log \hat{f}(\omega)$ may be returned, with additive limits.

The bandwidth b of the corresponding smoothing window in the frequency domain is also provided. Spectrum estimates separated by (angular) frequencies much greater than b may be assumed to be independent.

4. Parameters

nx

Input: the length of the time series, n.
Constraint: **nx** ≥ 1.

mt_correction

Input: whether the data are to be initially mean or trend corrected.
mt_correction = **Nag_NoCorrection** for no correction, **mt_correction** = **Nag_Mean** for mean correction, **mt_correction** = **Nag_Trend** for trend correction.
Constraint: **mt_correction** = **Nag_NoCorrection**, **Nag_Mean** or **Nag_Trend**

px

Input: the proportion of the data (totalled over both ends) to be initially tapered by the split cosine bell taper. (A value of 0.0 implies no tapering).
Constraint: $0.0 \leq$ **px** ≤ 1.0.

mw

Input: the value of M which determines the frequency width of the smoothing window as $2\pi/M$. A value of n implies no smoothing is to be carried out.
Constraint: $1 \leq$ **mw** $\leq$ **nx**.

pw

Input: the shape parameter, p, of the trapezium frequency window.

A value of 0.0 gives a triangular window, and a value of 1.0 a rectangular window.

If **mw** = **nx** (i.e., no smoothing is carried out), then **pw** is not used.
Constraint: $0.0 \leq$ **pw** ≤ 1.0. if **mw** $\neq$ **nx**.

l

Input: the frequency division, L, of smoothed spectral estimates as $2\pi/L$.
Constraints: **l** ≥ 1
l must be a factor of **kc** (see below).

kc

Input: the order of the fast Fourier transform (FFT), K, used to calculate the spectral estimates. **kc** should be a multiple of small primes such as 2^m where m is the smallest integer such that $2^m \geq 2n$, provided $m \leq 20$.

Constraints: **kc** $\geq 2 \times$ **nx**,
kc must be a multiple of **l**.

The largest prime factor of **kc** must not exceed 19, and the total number of prime factors of **kc**, counting repetitions, must not exceed 20. These two restrictions are imposed by nag_fft_real (c06eac) which performs the FFT.

lg_spect

Input: indicates whether unlogged or logged spectral estimates and confidence limits are required.

lg_spect = **Nag_Unlogged** for unlogged. **lg_spect** = **Nag_Logged** for logged.

Constraint: **lg_spect** = **Nag_Unlogged** or **Nag_Logged**.

x[kc]

Input: the n data points.

g

Output: vector which contains the **ng** spectral estimates $\hat{f}(\omega_i)$, for $i = 0, 1, \ldots, [L/2]$, in **g**[0] to **g**[**ng**-1] (logged if **lg_spect** = **Nag_Logged**). The memory for this vector is allocated internally. If no memory is allocated to **g** (e.g. when an input error is detected) then **g** will be NULL on return. If repeated calls to this function are required then **NAG_FREE** should be used to free the memory in between calls.

ng

Output: the number of spectral estimates, $[L/2] + 1$, in **g**.

stats[4]

Output: four associated statistics. These are the degrees of freedom in **stats**[0], the lower and upper 95% confidence limit factors in **stats**[1] and **stats**[2] respectively (logged if **lg_spect** = **Nag_Logged**), and the bandwidth in **stats**[3].

fail

The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

NE_BAD_PARAM

On entry, parameter **lg_spect** had an illegal value.
On entry, parameter **mt_correction** had an illegal value.

NE_INT_ARG_LT

On entry, **nx** must not be less than 1: **nx** = ⟨*value*⟩.
On entry, **mw** must not be less than 1: **mw** = ⟨*value*⟩.
On entry, **l** must not be less than 1: **l** = ⟨*value*⟩.

NE_REAL_ARG_LT

On entry, **px** must not be less than 0.0: **px** = ⟨*value*⟩.
On entry, **pw** must not be less than 0.0: **pw** = ⟨*value*⟩.

NE_REAL_ARG_GT

On entry, **px** must not be greater than 1.0: **px** = ⟨*value*⟩.
On entry, **pw** must not be greater than 1.0: **pw** = ⟨*value*⟩.

NE_2_INT_ARG_GT

On entry, **mw** = ⟨*value*⟩while **nx** = ⟨*value*⟩. These parameters must satisfy **mw** ≤ **nx**.

NE_2_INT_ARG_CONS

On entry, **kc** = ⟨*value*⟩while **nx** = ⟨*value*⟩. These parameters must satisfy **kc** ≥ 2***nx** when **nx** > 0.
On entry, **kc** = ⟨*value*⟩while **l** = ⟨*value*⟩. These parameters must satisfy **kc**%**l**== 0 when **l** > 0.

NE_FACTOR_GT
At least one of the prime factors of **kc** is greater than 19.

NE_TOO_MANY_FACTORS
kc has more than 20 prime factors.

NE_SPECTRAL_ESTIM_NEG
One or more spectral estimates are negative. Unlogged spectral estimates are returned in **g** and the degrees of freedom, unlogged confidence limit factors and bandwith in **stats**.

NE_CONFID_LIMIT_FACT
The calculation of confidence limit factors has failed. Spectral estimates (logged if requested) are returned in **g**, and degrees of freedom and bandwith in **stats**.

NE_ALLOC_FAIL
Memory allocation failed.

NE_INTERNAL_ERROR
An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

6. Further Comments

nag_tsa_spectrum_univar carries out a FFT of length **kc** to calculate the sample spectrum. The time taken by the routine for this is approximately proportional to **kc** $\times$ log (**kc**) (see nag_fft_real (c06eac) for further details).

6.1. Accuracy

The FFT is a numerically stable process, and any errors introduced during the computation will normally be insignificant compared with uncertainty in the data.

6.2. References

Bloomfield P (1976) *Fourier Analysis of Time Series: an Introduction.* Wiley.
Jenkins G M and Watts D G (1968) *Spectral Analysis and its Applications.* Holden-Day.

7. See Also

None.

8. Example

The example program reads a time series of length 131. It selects the mean correction option, a tapering proportion of 0.2, the option of no smoothing and a frequency division for logged spectral estimates of $2\pi/100$. It then calls nag_tsa_spectrum_univar to calculate the univariate spectrum and prints the logged spectrum together with 95% confidence limits. The program then selects a smoothing window with frequency width $2\pi/30$ and shape parameter 0.5 and recalculates and prints the logged spectrum and 95% confidence limits.

8.1. Program Text

```
/* nag_tsa_spectrum_univar(g13cbc) Example Program.
 *
 * Copyright 1996 Numerical Algorithms Group.
 *
 * Mark 4, 1996.
 *
 */

#include <nag.h>
#include <nag_stdlib.h>
#include <stdio.h>
#include <nagg13.h>

#define KCMAX 400
#define NXMAX KCMAX/2
```

```
main()
{

  double stats[4];
  double x[KCMAX], xh[NXMAX], *g;
  double pw, px;

  Integer i, l;
  Integer kc, ng;
  Integer mw, nx;

  Vprintf("g13cbc Example Program Results\n");
  /*     Skip heading in data file */
  Vscanf("%*[^\n] ");

  Vscanf("%ld ",&nx);
  if (nx > 0 && nx <= NXMAX)
    {
      for (i = 1; i <= nx; ++i)
        Vscanf("%lf ", &xh[i - 1]);
      px = .2;
      mw = nx;
      pw = .5;
      kc = KCMAX;
      l = 100;
      while ((scanf("%ld ", &mw)) != EOF)
        {
          if (mw > 0 && mw <= nx)
            {
              for (i = 1; i <= nx; ++i)
                x[i - 1] = xh[i - 1];

              g13cbc(nx, Nag_Mean, px, mw, pw, l, kc, Nag_Logged, x, &g, &ng, stats,
                     NAGERR_DEFAULT);

              if (mw == nx)
                Vprintf("\n No smoothing\n\n");
              else
                Vprintf("\n Frequency width of smoothing window = 1/%ld\n\n", mw);
              Vprintf(" Degrees of freedom =%4.1f      Bandwidth =%7.4f\n\n",
                      stats[0],stats[3]);
              Vprintf(" 95 percent confidence limits -     Lower =%7.4f  \
Upper =%7.4f\n\n", stats[1], stats[2]);
              Vprintf("       Spectrum       Spectrum       Spectrum\
       Spectrum\n");
              Vprintf("       estimate       estimate       estimate\
       estimate\n\n");
              for (i = 1; i <= ng; ++i)
                Vprintf("%5ld%10.4f%s",i,g[i - 1], (i%4==0 ? "\n": ""));
              Vprintf("\n");
              if (g)
                NAG_FREE(g);
            }
        }
    }
  exit(EXIT_SUCCESS);
}
```

8.2. Program Data

```
g13cbc Example Program Data
131
11.500  9.890  8.728  8.400  8.230  8.365  8.383  8.243
 8.080  8.244  8.490  8.867  9.469  9.786 10.100 10.714
11.320 11.900 12.390 12.095 11.800 12.400 11.833 12.200
12.242 11.687 10.883 10.138  8.952  8.443  8.231  8.067
 7.871  7.962  8.217  8.689  8.989  9.450  9.883 10.150
10.787 11.000 11.133 11.100 11.800 12.250 11.350 11.575
11.800 11.100 10.300  9.725  9.025  8.048  7.294  7.070
```

```
 6.933  7.208  7.617  7.867  8.309  8.640  9.179  9.570
10.063 10.803 11.547 11.550 11.800 12.200 12.400 12.367
12.350 12.400 12.270 12.300 11.800 10.794  9.675  8.900
 8.208  8.087  7.763  7.917  8.030  8.212  8.669  9.175
 9.683 10.290 10.400 10.850 11.700 11.900 12.500 12.500
12.800 12.950 13.050 12.800 12.800 12.800 12.600 11.917
10.805  9.240  8.777  8.683  8.649  8.547  8.625  8.750
 9.110  9.392  9.787 10.340 10.500 11.233 12.033 12.200
12.300 12.600 12.800 12.650 12.733 12.700 12.259 11.817
10.767  9.825  9.150
131
30
```

8.3. Program Results

```
g13cbc Example Program Results

 No smoothing

 Degrees of freedom = 2.0      Bandwidth = 0.0480

 95 percent confidence limits -     Lower =-1.3053  Upper = 3.6762

       Spectrum       Spectrum       Spectrum       Spectrum
       estimate       estimate       estimate       estimate

    1   -5.9354    2   -0.1662    3   -0.8250    4   -0.9452
    5    3.2137    6    0.2738    7   -1.0690    8   -1.0401
    9   -1.2388   10   -3.5434   11   -5.2568   12   -3.2450
   13   -2.4294   14   -3.9987   15   -2.9853   16   -4.6631
   17   -4.3317   18   -4.6982   19   -4.6335   20   -3.6732
   21   -5.8411   22   -4.7727   23   -3.9747   24   -4.8351
   25   -5.9979   26   -6.1169   27   -5.5245   28   -4.4774
   29   -5.6331   30   -4.0707   31   -4.6921   32   -5.6515
   33   -9.2919   34   -4.6302   35   -4.1700   36   -4.7829
   37   -6.6058   38   -5.8145   39   -5.2714   40   -5.8736
   41  -10.2188   42   -5.7887   43   -7.0751   44   -7.4055
   45   -8.2774   46   -7.8966   47   -6.4435   48   -5.7844
   49   -5.4690   50   -6.8709   51   -8.7123

 Frequency width of smoothing window = 1/30

 Degrees of freedom = 7.0      Bandwidth = 0.1767

 95 percent confidence limits -     Lower =-0.8275  Upper = 1.4213

       Spectrum       Spectrum       Spectrum       Spectrum
       estimate       estimate       estimate       estimate

    1   -0.1776    2   -0.4561    3   -0.1784    4    1.9042
    5    2.1094    6    1.7061    7   -0.7659    8   -1.4734
    9   -1.5939   10   -2.1157   11   -2.9151   12   -2.7055
   13   -2.8200   14   -3.4077   15   -3.8813   16   -3.6607
   17   -4.0601   18   -4.4756   19   -4.2700   20   -4.3092
   21   -4.5711   22   -4.8111   23   -4.5658   24   -4.7285
   25   -5.4386   26   -5.5081   27   -5.2325   28   -5.0262
   29   -4.4539   30   -4.4764   31   -4.9152   32   -5.8492
   33   -5.5872   34   -4.9804   35   -4.8904   36   -5.2666
   37   -5.7643   38   -5.8620   39   -5.5011   40   -5.7129
   41   -6.3894   42   -6.4027   43   -6.1352   44   -6.5766
   45   -7.3676   46   -7.1405   47   -6.1674   48   -5.8600
   49   -6.1036   50   -6.2673   51   -6.4321
```